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TWO-PHASE UPFLOW IN RECTANGULAR CHANNELS

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Abstract—A comparison of a theoretical and experimental investigation of a two-phase concurrent flow of air and water in a vertical rectangular channel has been obtained. The model shows the importance of considering a lateral interfacial term in a fully developed flow situation. The one parameter model shows also that the extended idea of an "eddy diffusivity" as used in a single phase flow is feasible to model shear stress in the continuous phase.

INTRODUCTION

The use of sophisticated two-phase flow models has increased in the last ten years due to the advent of powerful computers. These models are in general two and multidimensional transient models that include a variety of the two-phase flow phenomena such as nonequilibrium thermal effects and unequal flow velocities Rivard (1977), Hirt (1979), Duval (1980). The constitutive relationships in these models play a very important role in determining how accurate their predictions could be. The need for improving our understanding of these constitutive relationships is still of keen interest to many researchers in this field.

In these cases, where one can assume the existence of "fully developed" flow the use of these codes would seem unjustified as the resulting equations are considerably simplified. These cases would be internal two-phase flows where the flow is adiabatic with no appreciable pressure drop through the system compared to the average system pressure in the case of a compressible gas. These requirements are needed in addition to the basic requirements of constant cross-section normal to the direction of flow and enough developing length to achieve the fully developed condition.

The purpose of this paper is to compare the results of two-phase flow experiments, described by Moujaes (1980) in a rectangular channel of 12.7 × 76.2 mm cross section, and such a model where the effect of a lateral lift force and the "turbulent eddy diffusivity" concept are considered.

CONSERVATION EQUATIONS

The conservation equations of mass, x momentum (axial), and y momentum (76.2 mm) for each phase are now given. It is assumed that the variation of velocity across the small spacing of the channel has been averaged. The momentum equations in particular must be given in great detail since the forces responsible for maintaining the void profile are often neglected. Only the steady-state version of these equations will be presented. The continuity equation for phase k is

$$\frac{\delta}{\delta x}(\rho_k \epsilon_k u_k) + \frac{\delta}{\delta y}(\rho_k \epsilon_k v_k) = 0, \qquad [1]$$

where $k = G$ for gas and L for liquid, and where ρ designates density and ϵ is the void fraction.

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The x and y momentum equations are

$$\frac{\delta}{\delta x}(\rho_k \epsilon_k u_k^2) + \frac{\delta}{\delta y}(\rho_k \epsilon_k u_k v_k) = -\frac{\delta}{\delta x}(\epsilon_k P) + \rho_k \epsilon_k g_x + \frac{\delta}{\delta x}(\epsilon_k \tau_{xx,k}) + \frac{\delta}{\delta y}(\epsilon_k \tau_{xy,k}) + F_{ix,k} - F_{wx,k} \qquad [2]$$

$$\frac{\delta}{\delta x}(\rho_k \epsilon_k u_k v_k) + \frac{\delta}{\delta y}(\rho_k \epsilon_k v_k^2) = -\frac{\delta}{\delta y}(\epsilon_k P) + \rho_k \epsilon_k g_k + \frac{\delta}{\delta x}(\epsilon_k \tau_{xy,k}) + \frac{\delta}{\delta y}(\epsilon_k \tau_{yy,k}) + F_{iy,k} - F_{wy,k} \qquad [3]$$

where u and v are the velocity components in the x and y directions (x direction is main direction of flow), and τ represents either shear or normal stresses with F designating either interfacial or wall shear respectively and i-denoting interface. It is now desirable to simplify these equations by making a number of assumptions. The most important assumption is that the flow becomes fully developed. The assumptions are:

1. Gas and liquid densities are constants.
2. Steady-state and fully developed flow, i.e. x-direction pressure gradient is a constant and all other partial derivatives with respect to x are zero.
3. Shear stress in the dispersed phase, i.e. gas, can be neglected.
4. Variations in the bulk normal stresses in the lateral direction are negligible.
5. Profiles are symmetric about the channel centerline and gravity acts only in the negative x direction.
6. Only the liquid phase wets the wall, i.e. wall drag is zero for gas.

With these assumptions, the continuity equation for each phase are identically satisfied. Furthermore, the phase velocities have the following form: $v_k = 0$, $u_k = u_k(y)$. However, the overall balance of the volume flow rate of each phase in the channel must be equal to the rate injected at the entrance of the channel. This establishes an important relationship between the velocity and void profiles, i.e.

$$Q_k = 2h \int_0^{y_c} \epsilon_k u_k \, \mathrm{d}y, \qquad [4]$$

where Q is the total volumetric flow rate, h is the depth of channel and y_c is half channel width. Because the transverse velocity components are identically zero and the axial phase velocities are only functions of y, the left-hand side of the momentum equations are zero. Thus, the x and y momentum equation have now been reduced to the following form:

$$0 = -\frac{\delta}{\delta x}(\epsilon_k p) + \rho_k \epsilon_k g_x + F_{ix,k} - F_{wx,k} + \frac{\delta}{\delta y}(\epsilon_k \tau_{xy,k}), \qquad [5]$$

$$0 = \frac{\delta}{\delta y}(\epsilon_k P) + F_{iy,k}. \qquad [6]$$

In [5], the last two terms drop out for the gas phase.

The compatibility relation between the two phases must hold, i.e.

$$\epsilon_G + \epsilon_L = 1.0. \qquad [7]$$

Also, the interfacial forces must balance between the two phases, i.e.

$$F_{ix,G} = -F_{ix,L}, \tag{8}$$

$$F_{iy,G} = -F_{iy,L}. \tag{9}$$

Another assumption must now be added to the model. This assumption is that there is negligible pressure drop across the liquid–gas interface. This is a reasonable assumption since nearly all bubbles are greater than 2 mm in diameter and the gas slugs are considerably larger that the bubbles.

Adding together [6] for both phases and using the compatibility relation gives

$$\delta p/\delta y = 0. \tag{10}$$

From this result, the lateral momentum equation for the gas phase becomes

$$0 = -p\frac{d\epsilon_G}{dy} + F_{iy,G}. \tag{11}$$

Under the assumption of negligible change in curvature and surface tension, Ishii (1975) presents a general expression for this interfacial force which is related to the individual processes causing this force. His expression is

$$F_{iy,G} = F_{drag,y} + p_i\frac{d\epsilon_G}{dy}, \tag{12}$$

where

$F_{drag,y}$ the net form and shear drag at the interface in y direction,
p_i the interfacial pressure (which is assumed different from the bulk pressure).

It at first may appear inconsistent that an interfacial pressure different from the bulk pressure may be important. However, if one visualizes bubbles as solid spheres, ordinary fluid mechanics shows that the pressure at the surface of the sphere is different from the bulk pressure.

Combining [11] and [12] gives

$$(p_i - p)\frac{d\epsilon_G}{dy} = -F_{drag,y}. \tag{13}$$

The final equation needed to complete the system is obtained by adding the x-momentum equations for each phase together. The interfacial force terms cancel out. For air–water flows at low pressure, the gas density is negligible compared to the liquid density and can be removed from the body-force term. The result is

$$-\frac{dp}{dx} + \epsilon_L\rho_L g_x + \frac{d}{dy}(\epsilon_L\tau_{xy,L}) + F_{wx,L} = 0. \tag{14}$$

Thus, there are five basic unknowns: ϵ_G, ϵ_L, u_G, u_L and p. There are five basic equations: [4] for each phase and [7], [13] and [14]. The volume flow rates Q_k are parameters and constitutive relations are needed for $F_{drag,y}$, $p_i - p$, $\tau_{xy,L}$, and $F_{wx,L}$.

CONSTITUTIVE EQUATIONS

A set of constitutive equations will now be developed which will allow a solution to the equation set just obtained. The origin of the model used was work by Delhaye (1969). Work of Herringe & Davis (1976) have shown that for practical purposes, the flow could be considered as fully developed for $L/D_h = 30$. In this work, L/D_h is approximately equal to 60 which amply satisfies their criteria.

Lateral interfacial drag

Using an analogy based on the work of Saffman (1965), Delhaye (1969) proposed a relation for the lateral interfacial force on a bubble. He assumed that this force had the following properties:

- Proportional to relative velocity gradient.
- Inversely proportional to turbulent eddy diffusivity of liquid.
- Inversely proportional to relative velocity.
- Proportional to liquid kinematic viscosity of liquid.

Hence, the following relation was obtained:

$$\frac{F_{\text{bubble},y}}{\rho_L(u_G - u_L)^2 d^2} = C \frac{\upsilon_L d}{e(u_G - u_L)} \frac{\mathrm{d}(u_G - u_L)}{\mathrm{d}y}, \qquad [15]$$

where $F_{\text{bubble},y}$ designates the interfacial force on the bubble, d diameter for the bubble, C constant of proportionality, υ_L is the liquid kinematic viscosity and e is the turbulent eddy diffusivity for transverse lift term.

Equation [15] is basically a lateral "lift" coefficient. Now, the void fraction is used to relate individual bubbles to the net lateral drag on the gas phase. The result is

$$F_{\text{drag},y} = F_{\text{bubble},y}(6\epsilon_G/\pi d^3) \qquad [16]$$

Combining [15] and [16] gives

$$F_{\text{drag},y} = \left(\frac{6C}{\pi}\right) \frac{\epsilon_G \upsilon_L(u_G - u_L)}{e} \frac{\mathrm{d}(u_G - u_L)}{\mathrm{d}y}. \qquad [17]$$

The turbulent eddy diffusivity used in [15] and [17], in analogy with the study by Saffman (1965), can be related to the relative velocity using the standard mixing length approach:

$$e = -k^2(y_c - y)^2 \frac{\mathrm{d}(u_G - u_L)}{\mathrm{d}y}, \qquad [18]$$

where k^2 is a constant.

Equation [18] is substituted into [17] to eliminate the velocity gradient multiplier. The result is

$$F_{\text{drag},y} = -\left(\frac{6C\upsilon_L}{\pi k^2}\right) \frac{\epsilon_G(u_G - u_L)}{(y_c - y)^2}. \qquad [19]$$

This equation gives the form of the lateral force on the gas phase. The scaling constant C remains to be determined.

Validation of Saffman's (1965) term needs three conditions to be satisfied. The first two of these conditions are satisfied in this study. Saffman (1965) points out that observations made by Oliver (1962) show qualitative agreement with that study in a Poiseuille flow.

These observations back the assumption used in this study of the existence of such a lateral drift term.

Liquid shear stress

The next relation to be considered is the shear stress in the liquid phase. Again, a turbulent viscosity approach will be taken with the turbulent diffusivity (e_L) of the liquid given by a mixing length model. Hence,

$$\tau_{xy,L} = e_L \frac{du_L}{dy}, \quad [20]$$

$$e_L = -k^2(y_c - y)^2 \frac{du_L}{dy}. \quad [21]$$

Equations [18] and [21] are similar. Both use the same mixing length constant k^2 (mainly due to lack of any better information). However, the gradients are different because the relative velocity determine forces acting on the gas phase while the variation of the liquid velocity determines shear forces in the liquid phase alone. In a rectangular channel secondary flows exist near the corners. The literature available to the authors does not present the effect of this flow on the diffusivity. However as will be seen from the reasonable agreement between theory and experiment, the present expression is adequate.

Wall drag forces

The flow under consideration is that of a gas–liquid flow which is flowing upward (x direction) with variation of velocity and void in the transverse direction (y direction). The flow is confined between parallel plates (z direction). The velocities and void profiles to be obtained are the averages for the z direction. The confining plates impact on the flow by introducing drag forces on the liquid phase as indicated in [14]. If variation of the frictional drag with y is neglected as a first approximation, then conventional wall drag relations can be used. Data from Jones (1973) shows that with an aspect ratio of 10:1 in a rectangular geometry, one finds significant variations of void fraction along the y direction. The literature shows that aspect ratios 20:1 and up are considered adequate for one dimensional flow analysis. The significance of the variations in the y direction is shown in the experimental results. Thus,

$$F_{wx,L} = \phi_{f_0}^2 C_d (G_L^2/2\rho_L)/D_h, \quad [22]$$

where ϕ_f^2 is the two-phase multiplier, G_L designates liquid mass flow rate and D_h hydraulic diameter.

Wallis (1969) indicates that a value of $C_d = 0.005$ (friction factor) is reasonably accurate for a wide range of air–water flows. Data of Nakoryakov *et al.* (1981), have shown that for $\beta > 0.25$, which is the case in this study, agreement exists between their data and the correlation of Armand (1950) in regard to the value of the two-phase wall shear. The two phase multiplier for low-quality gas–liquid flows is

$$\phi_f^2 = 1 + x(\rho_L/\rho_G - 1), \quad [23]$$

where x is the gas quality.

This approximation to the friction term applies to the shear stress in the z direction.

Flow regime considerations

A constitutive relation is still needed for the relative velocity used in [19] and in calculating the gas velocity profile from the liquid velocity profile. The drift-flux model of

Zuber and Findlay (1965) can be used to supply this relation. The relation between relative velocity and drift velocity can be represented as

$$u_{rel} = u_G - u_L = \{(C_0 - 1)j + u_{Gj}\}/\epsilon_L, \tag{24}$$

where u_{Gj} is the drift flux velocity of the gas and C_0 the distribution parameter of two-phase flow. The expression u_{rel} used in [24] relate to the average relative velocity between the phases. In this work it will be used as an expression for the local relative velocity and varies with ϵ_L across the channel in the absence of any more suitable correlations for this term. This term has the right trend in that u_{rel} increases as one approaches the core because ϵ_L usually decreases, while the other terms in this expression are constant, as the axis is approached.

Zuber and Findlay have shown that the drift velocities are functions of flow regime. They present the following relations for round tubes:

Bubbly flow regime:

$$u_{Gj} = 1.5(\sigma g/\rho_L)^{1/4} \tag{25}$$

σ = surface tension.

Slug flow regime:

$$u_{Gj} = 0.35(gD_h)^{1/2}. \tag{26}$$

The values of C_0 of 1.23 and 1.2 were used from Jones (1973) for bubbly and slug flow respectively.

SOLUTION OF EQUATION SET

The solution of the equation set will now be outlined. First, an expression for the void profile will be obtained. Equation [24] is substituted into [19] and the resulting expression is substituted into [13]. After rearrangement, the result is

$$\frac{d\epsilon_G}{dy} = -\frac{A\{(C_0 - 1)_j + u_{Gj}\}}{(y_c - y)^2}\frac{\epsilon_G}{1 - \epsilon_G}, \tag{27}$$

where $A = 6C\mu_L/\pi k^2(p - p_i)$; $T = \{(C_0 - 1)_j + u_{Gj}\}$, where μ_L is dynamic liquid viscosity. Equation [27] is integrated by first nondimensionalizing the distance from the axis of symmetry. Thus,

$$s = 1 - y/y_c. \tag{28}$$

Then, an analytical solution for ϵ_G is obtained, i.e.

$$\ln\left(\frac{\epsilon_G}{\epsilon_{GC}}\right) + \epsilon_{GC} - \epsilon_G = \frac{T}{y_c}\left(1 - \frac{1}{s}\right). \tag{29}$$

This equation satisfies the requirement that ϵ_G approaches zero as s approaches zero, i.e. at the wall. The solution does not give zero void gradient near the center of the channel. However, the void distribution is quite flat in this region. The constant A contains the unknown parameters C and $(p - p_i)$, and, hence, must be determined by matching the data. The centerline gas void fraction ϵ_{GC} will be determined by matching the gas and liquid flow rates given in [4].

In order to obtain an expression for the liquid velocity, the shear relationship of [20] and [21] plus the wall drag relations of [22] and [23] are substituted into [14]. A differential

Table 1. Experimental conditions

Run No.	M_L (kg/s)	M_G (kg/s)	Regime	Temp. (°C)	System (gage) Pressure (N/cm^2)
1	0.68	7.1×10^{-4}	Bubbly	24.0	1.745
2	0.73	12.5×10^{-4}	Slug	24.0	2.233
3	0.78	17.1×10^{-4}	Slug	23.5	3.065
4	0.92	22.6×10^{-4}	Slug	25.0	4.188
5	0.91	4.5×10^{-4}	Bubbly	24.0	2.213
6	0.83	6.4×10^{-4}	Bubbly	24.0	1.745

equation for the liquid velocity of the following form is obtained:

$$\epsilon_L k^2 (y_c - y)^2 \left(\frac{du_L}{dy}\right)^2 = \int_0^y \left[-\frac{dp}{dz} + \epsilon_L \rho_L g_x + F_{wx,L} \right] dy \qquad [30]$$

with the boundary condition that $u_L = 0$ when $y = y_c$. This equation can be solved for u_L if values of the pressure gradient are given.

The gas velocity distribution can be obtained from [24] once values of liquid velocity and void profile are known. Thus,

$$u_G = u_L + \{(C_0 - 1)_j + U_{Gj}\}/\epsilon_L. \qquad [31]$$

The mixing length constant k^2 was set equal to 0.08. The value of A is 2.57×10^{-3} s. This left two unknown parameters ϵ_{GC} and dp/dx in [2] through [31]. An iteration was performed by varying these two parameters until the flow rates calculated by [4] matched the experimental values as measured by the total flow measuring meters.

RESULTS AND DISCUSSION

Six sets of experimental data for the vertical channel were obtained by Moujaes (1980). The test conditions and flow regimes are indicated in table 1. These six data sets are equally divided between the bubbly and slug flow regimes. Table 2 indicates the values of the parameters selected in obtaining the theoretical velocity and void distributions. Since the pressure gradient was one of the selected parameters, this quantity could be compared to the measured value. This comparison is also shown in table 2.

Figures 1 through 6 show the resulting plots of void fraction, liquid velocity, and gas velocity for the six sets of data. It is important to note that the theoretical values were selected to agree with liquid and gas flow rates as measured by flow meters at the entrance to the test section. The experimental results integrated over the flow area are between 5% and 20% below the values determined by the flowmeters. This difference is related to the accuracy of the void and velocity measurements and accounts for the major discrepency between the theoretical and experimental curves. Hence, if the experimental velocities were slightly increased, better agreement would be obtained with the theoretical model.

Table 2. Theoretical results

Run No.	Center void ϵ_{GC}	Theoretical press. drop N/(cm^3)	Experimental pressure drop (N/cm^3)	Factor A (s)
1	0.358	6.9×10^{-3}	6.6×10^{-3}	2.57×10^{-3}
2	0.464	6.3×10^{-3}	5.7×10^{-3}	2.57×10^{-3}
3	0.514	6.1×10^{-3}	5.4×10^{-3}	2.57×10^{-3}
4	0.535	6.3×10^{-3}	5.9×10^{-3}	2.57×10^{-3}
5	0.209	8.4×10^{-3}	7.9×10^{-3}	2.57×10^{-3}
6	0.297	7.6×10^{-3}	6.6×10^{-3}	2.57×10^{-3}

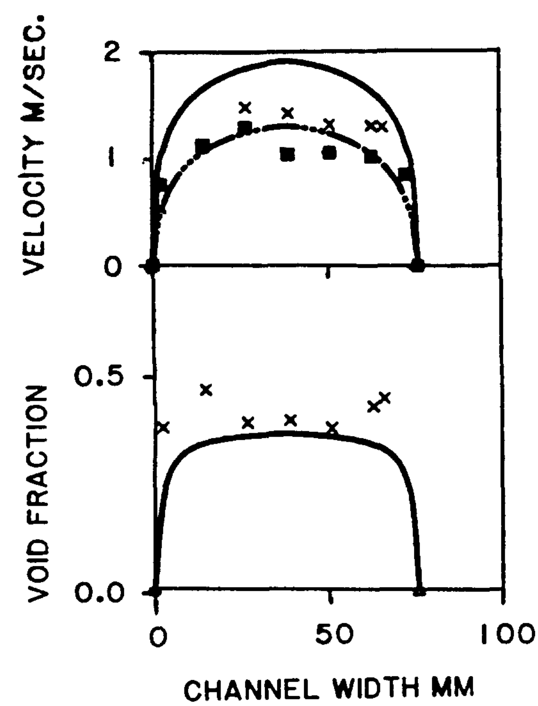

Figure 1. Bubbly flow. × Exp. data void fraction or gas velocity, ■ Exp. data liquid velocity, —— Prediction void fraction or gas velocity, —· · ·— Prediction liquid velocity.

Unfortunately, not enough experimental data are available to describe in more detail the $P - P_i$ term due to the complex physical behavior involved when the flow fields of other bubbles interfere with a bubble in question which make an analytical description of the term impossible.

Malnes (1966), Nakoryakov & Kashinsky (1981), and Herringe & Davis (1976) have reported off-center peaking of void fraction profiles in bubbly flows. This work also shows

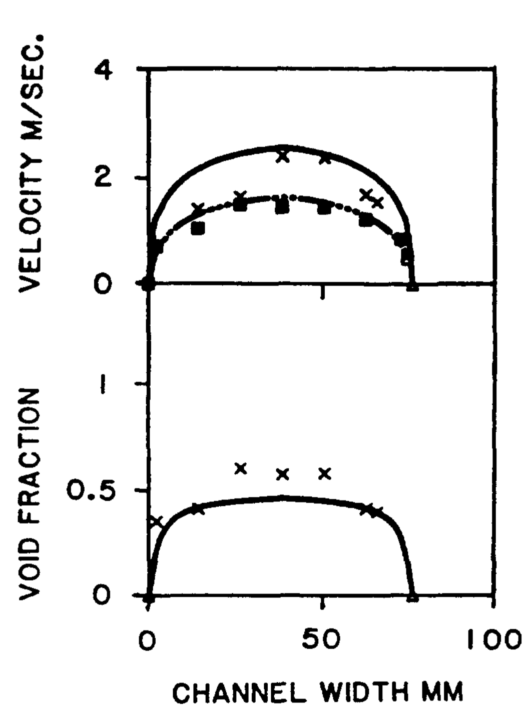

Figure 2. Slug flow (for legend see figure 1).

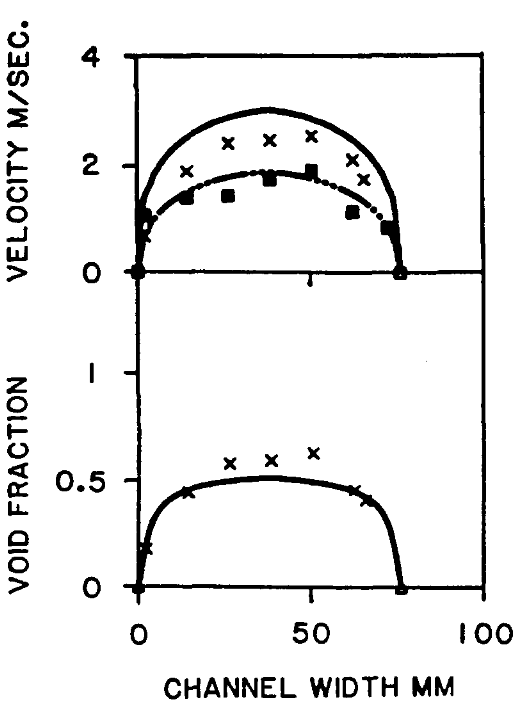

Figure 3. Slug flow (for legend see figure 1).

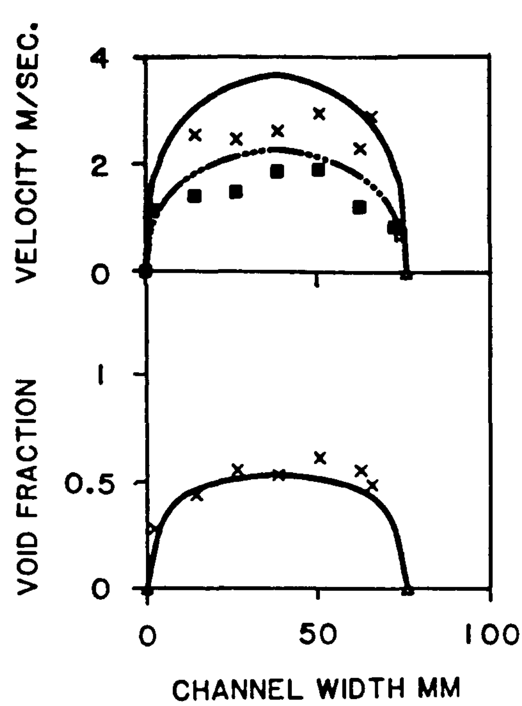

Figure 4. Slug flow (for legend see figure 1).

similar experimental trends in runs 1, 5, and 6. Rouhani (1976) suggested that this phenomenon could be as a result of vortex generation near the walls due to shear effects. This vortex generation in turn traps the smaller bubbles near the locus of the center of rotation of the vortices. The action persists until the two off-center peaks are bridged by the formation of large slugs (i.e. slug-flow regime) at higher voidage. The present model does not predict this localized behavior, but indicates a monotonic increase in voidage more typical of the core of the flow.

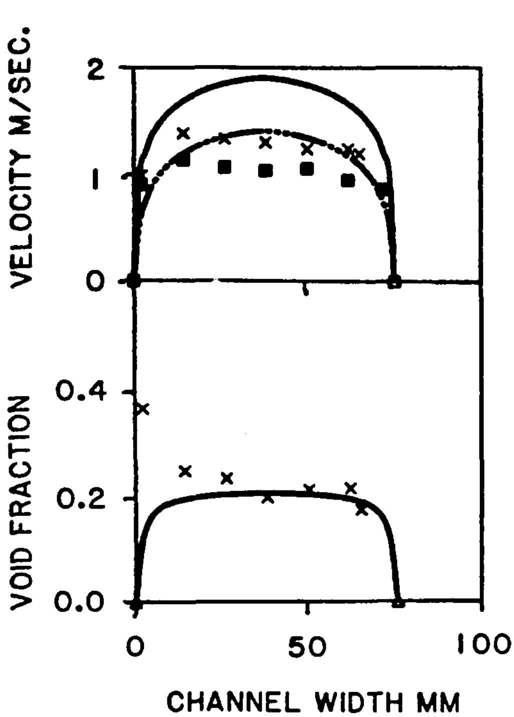

Figure 5. Bubbly flow (for legend see figure 1).

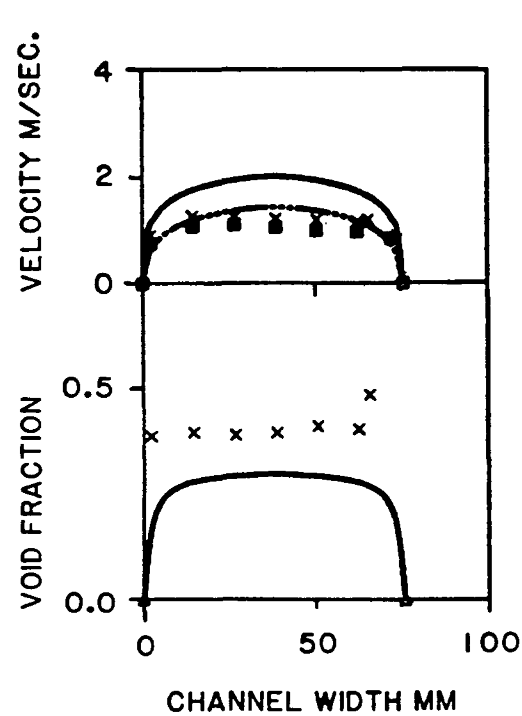

Figure 6. Bubbly flow (for legend see figure 1).

In the slug flow data sets, there are no off-center peaks as expected and the model seems to show better agreement with the data.

Another result of this work is that the use of an "eddy viscosity" concept to calculate the continuous phase turbulent relations seems to work reasonably well. No local values of gas or liquid velocities could be obtained very close to the wall. Nevertheless, these are expected to be similar to single-phase flow with high gradients near the walls. The experimental data supports this conclusion as far as it extends into the near-wall region.

Nakoryakov & Kashinsky (1981) have recently obtained measurements in this region as did Sato & Sadatomi (1981) and have presented an analysis similar to the results of this section. However, their analysis required the void profile as input. Lahey (1978) has also studied the bubbly void fraction for the purpose of predicting near wall void peaking. He used the concept of an "inner" and "outer" solution to the axial momentum equation. However, the means of joining the two solutions still relies on empirical fitting of the results.

The total pressure drop predicted by the model differs from the measured one between 2%–10% which is considered satisfactory and not surprising because all these flow situations are still gravity dominated two-phase flow phenomena.

The present approach has one feature that is unique from those just mentioned. That feature is it being used to model two-phase flow regime and predict the four sought quantities simultaneously (dP/dx), ϵ, u_L, and from it u_G.

More work is needed to check the model against the data of Jones (1973) for void fraction and liquid velocity distributions in rectangular channels and with Herringe & Davis (1976) for void fraction and gas velocity distributions in circular tubes.

NOMENCLATURE

u = velocity in x direction
v = velocity in y direction
k = Phase; G for gas and L for liquid
F = force, interfacial, wall friction
P = system pressure
j = total volumetric flux
g = acceleration due to gravity
h = channel half width
Q = volumetric flux rate
C = proportionality constant
C_d = single-phase drag coefficient
G_L = liquid mass velocity
d = bubble diameter
y_c = half distance transverse to main flow direction
D_h = hydraulic diameter of channel
x = gas quality
C_o = flow distribution parameter
L = axial length of the experimental data plane from two-phase flow entry.

Greek symbols

ϵ = void fraction gas or liquid
τ = shear stress
e = turbulent eddy diffusivity for transverse lift term
e_L = turbulent eddy diffusivity for liquid shear stress
ϕ_f^2 = two-phase multiplier

ρ = density
σ = surface tension
β = ratio of gas volume flux to total volume flux

REFERENCES

DELHAYE, J. M. 1969 General Equations of Two-Phase Systems and Their Applications to Air–Water Bubble Flow and To Steam-Water Flashing Flow ASME publication 69-HT-63.

GRIFFITH, L. 1963 The Prediction of Low Quality Boiling Void. ASME preprint no. 63-HT-20, National Heat Transfer Conference, Boston, Mass.

HALL, C. A., PORSCHING, T. A. & DOUGALL, R. S. 1980 Numerical Methods for Thermally Expandable Two-Phase Flow-Computer Techniques for Steam Generator Modeling. EPRI Research Project 963-1.

HERRINGE, R. A. & DAVIS, M. R. 1976 Structural development of Gas Liquid Mixture Flows. *J. Fluid Mechan.* **73**, 97–123.

HIRT, C. W., ROMERO, N. C., TORREY, M. D. & TRAVIS, J. R. 1979 SOLA-DF A Solution Algorithm for Nonequilibrium Two-Phase Flow. (NUREG/CR-0690-LA-7725-MS.

ISHII, M. 1975 *Thermo-Fluid Dynamic Theory of Two-Phase Flow.* Eyrolles, Paris.

JONES, O. C. 1973 Statistical Considerations in Heterogenous Two-Phase Flowing Systems. PhD thesis, Rensselear Polytechnic Institute.

LAHEY, R. T., Jr. 1978 Two-Phase Flow Phenomena in Nuclear Reactor Technology. Rensselaer Polytechnic Institute, Troy, NY NUREG/CR-0702 Quarterly Progress Report No. 10 Sept.–Nov.

MALNES, D. 1966 Slip Ratios and Friction Factors in the Bubble Flow Regime in Vertical Tubes, KR-110.

MOUJAES, S. 1980 Cocurrent Two-Phase vertical flow in rectangular channels. PhD thesis, University of Pittsburgh.

NAKORYAKOV, V. E. & KASHINSKY, O. N. 1981 Local Characteristics of Upward Gas–Liquid Flows. *Int. J. Multiphase Flow* **7** 63.

NICKLIN, D. J., WILKES, J. O. & DAVIDSON, J. F. 1962 Two-Phase Flow in Vertical Tubes, *Trans. Instr. Chem. Engrs.* **40,** 61–68.

RIVARD, W. C & TORREY, M. D. 1977 K-FIX A Computer Program for Transient Two-Phase Two Dimensional, Two-Fluid Flow. LA-NUREG-6623.

ROUHANI, Z. 1976 Effect of Wall Friction and Vortex generation of the Radial Distribution of Different Phases. *Int. J. Multiphase Flow* **3**, 35–50.

SAFFMAN, P. G. 1965 The Lift on a small Sphere in a Slow Shear Flow. *J. Fluid Mechan.* **22,** 385–400.

SATO, J. & SADATOMI, M. 1981 Momentum and Heat Transfer in Two-Phase Bubbly Flow. I. *Int. J. Multiphase Flow* **7,** 167–179.

WALLIS, B. G. 1969, *One Dimensional Two-Phase Flow.* McGraw-Hill, Inc., New York.

ZUBER, N. & FINDLAY, J. A. 1965 Average Volumetric Concentration in Two-Phase Flow Systems, *J. Heat Transfer* **87,** 453–468.